1. [12] Give the definition of the derivative $f'(x)$ of a function $f(x)$, and use it to find $(\frac{1}{x})'$. 

2. [24] The position $s(t)$ of a point moving along a straight line is given by

$$s(t) = \frac{10t}{t^2 + 1}, \quad t \geq 0.$$ 

Given that $s'(t) = 10\frac{1-2t^2}{(t^2+1)^2}$ and $s''(t) = 20\frac{(t^2-3)}{(t^2+1)^3}$ answer the following:
(a) When does $s(t)$ reach its maximum and what is the value of this maximum?
(b) When is the velocity a maximum and what is the value of this maximum?
(c) What is the average velocity over the time interval $[0,2]$?
(d) What is the average acceleration over the time interval $[0,2]$?

3. [12] Find the equations of the tangent and the normal lines to the curve $y = x^3 - 2x$ at the point $(0,0)$.

4. [20] Evaluate the following limits. [Justify your answers.]

(a) $\lim_{x \to -2} \frac{\sqrt{x-1} - 1}{x - 2}$  
(b) $\lim_{x \to \pi/4} \frac{3 - 2x^2}{\sqrt{1 + 7x^2}}$  
(c) $\lim_{x \to -\infty} \frac{\sin x}{2x}$  
(d) $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} \sin x & \text{for } x > 0 \\ \frac{x}{1-x} & \text{for } x \leq 0 \end{cases}$.

5. [10] Use calculus to find the point on the line $y = 2x - 3$ closest to the point $(5, -3)$.

6. [25] Find the following:

(a) $\int_1^3 (x^{-2} + x^2) \, dx$  
(b) $\int x(1 - x^2)^{1/2} \, dx$  
(c) $\int \sin x \cos(x) \, dx$  
(d) $\int_0^{\pi/6} \tan^2 x \, dx$  
(e) $\int \frac{\sin x}{\cos^2 x} \, dx$.

7. [10] Find the area in the first quadrant between the curves $y = \sin x$ and $y = \frac{2}{\pi} x$. [You may use your calculator to help find where the curves intersect.]

8. [5] Given a polynomial $f(x)$. Proves that between any two consectutive roots of $f(x)$ there is a root of $f'(x)$. [Hint: Use Rolle’s theorem.]

9. [24] Differentiate the following:

(a) $f(x) = \cot(\sqrt{x})$  
(b) $f(x) = (x^2 + 1)^5(x^3 - 1)^7$  
(c) $f(x) = \frac{x^5}{x^2 + 1}$  
(d) $f(x) = \int_0^\pi \sin(t^2) \, dt$.

10. [6] Find $\frac{dy}{dx}$: $\sin(xy) + x^2 y^3 = 1$. 

11. [10 pts] A contractor agrees to paint 1000 circular signs of radius 3. Upon receiving the signs it is found that the radii are 3.01. Use differentials to estimate the additional area to be painted.

12. [20 pts] Given

\[ f(x) = \frac{x}{(x+1)^2} \quad \text{for} \quad x \neq -1 \]

and

\[ f'(x) = \frac{1-x}{(x+1)^3} \quad f''(x) + \frac{2(x-2)}{(x+1)^4}, \]

find

(a) the interval(s) on which \( f(x) \) is increasing/decreasing;
(b) the local max/min of \( f(x) \), if any;
(c) the intervals on which \( f(x) \) is concave up/down;
(d) any inflection points of \( f(x) \);
(e) the asymptotes of \( f(x) \).

(f) Sketch the graph of \( f(x) \) indicating the information in (a)–(e).

13. [10 pts] A child blows up a spherical balloon so that its volume is increasing at the rate of 40 cubic inches per minute. How fast is the surface area of the balloon increasing when the radius is 10 inches? [The volume of a sphere is \( V = \frac{4}{3} \pi r^3 \), while the surface area is \( S = 4\pi r^2 \).]

14. [12 pts] A rectangular beam is to be cut from a log with circular cross section and diameter 12. [See picture.] The strength of the beam is given by \( S = kxy^2 \) where \( k \) is a constant. Find the dimensions of the strongest beam which can be cut.